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**24-MA-21**

# M.Sc. II SEMESTER [MAIN/ATKT] EXAMINATION
## JUNE - JULY 2024

# MATHEMATICS
Paper - I
## [Advanced Abstract Algebra - II]

*[Max. Marks : 75]* *[Time : 3:00 Hrs.]* *[Min. Marks : 26]*

**Note :** Candidate should write his/her Roll Number at the prescribed space on the question paper.
Student should not write anything on question paper.
Attempt five questions. Each question carries an internal choice.
Each question carries **15 marks.**

**Q. 1 a)** Prove that the submodules of the quotient module M/N are of the form U/N, where U is a submodule of M containing N.

**b)** State and Prove fundamental theorem of R-homomorphisms.

**OR**

**a)** Let R be a ring with unity. Prove that an R-module M is cyclic if and only if $M \simeq R / I$ for some left ideal I of R.

**b)** Let A and B be R-submodules of R-modules M and N respectively. Then show that

$$\frac{M \times N}{A \times B} \simeq \frac{M}{A} \times \frac{N}{B}$$

**Q. 2 a)** State and prove Schur's lemma for simple R-module.

**b)** Let M be a free R-module with a basis $\{e_1, \ldots, e_n\}$. Then show that $M \simeq R^n$

**OR**

**a)** Prove that every finitely generated module is a homomorphic image of a finitely generated free module.

**b)** Let M be a finitely generated free module over a commutative ring R. then prove that all bases of M are finite.

**Q. 3** Prove that for an R-module M, the following are equivalent.

i) M is Noetherian.

ii) Every submodule of M is finitely generated.

iii) Every non empty set S of submodules of M has a maximal element.

**P.T.O.**

**OR**

**a)** Let M be an R-module and let N be an R-submodule of M. Then prove that M is noetherian if and only if both N and M/N are noetherian.

**b)** Show that a subring of a noetherian ring need not be noetherian.

**Q. 4 a)** Let M be a noetherian module or any module over a noetherian ring. Then show that each non zero submodule contains a uniform module.

**b)** Let M be a non zero finitely generated module over a commutative noetherian ring R. Then prove that there are only a finite number of primes associated with M.

**OR**

**a)** Let R be a principal ideal domain, and let M be an R-module. Then prove that

Tor M = $\{x \in M \mid x \text{ is torsion}\}$ is a submodule of M.

**b)** Find the abelian group generated by $(x_1, x_2, x_3)$ subject to

$$2x_2 - x_3 = 0$$
$$-3x_1 + 8x_2 + 3x_3 = 0$$
$$2x_1 - 4x_2 - x_3 = 0$$

**Q. 5** Find rational canonical form of the matrix A where

$$A = \begin{pmatrix} 2 & 4 & 0 \\ 1 & 4 & 1 \\ 3 & 8 & 3 \end{pmatrix}$$

**OR**

Prove that two nilpotent linear transformations are similar if and only if they have the same invariants.

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